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CORRELATION DETECTION OF SEISMIC DISTURBANCES

By

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ABSTRACT

In a medium in which noise and signal propagate with different velocities, a detection scheme making use of the cross-correlation between the received waveforms at two different locations offers a number of distinct advantages over one making use of ordinary autocorrelation of the received waveform. These advantages arise from the resultant relative shift between the noise and signal autocorrelation functions allowing for separation of noise and signal when the noise characteristics or statistics are unknown, and even where the processes are non-stationary. Ramifications of the correlation shift are an improvement in signal-to-noise ratio (even when noise characteristics are known), and capability for measurement of velocity, signal source direction, and signal and noise and channel characteristics.

PREFACE

With the current interest in detection of underground nuclear explosions, this report has been prepared to point out that effective use can be made of the dual-velocity properties of the medium. The report is a modification of a memorandum originally prepared in 1960 while at the Applied Science Division of Melpar, Inc., Watertown, Massachusetts. I would like to thank James Lee, presently of Raytheon Company, for pointing out that seismic signals propagate with different velocities and suggesting that two receivers might be used to exploit this property.

I. INTRODUCTION

Usual detection of a signal in noise involves cross-correlation when the expected received signal waveform is known, and auto-correlation when it is unknown, all at a single receiver. There are environments in which signal and noise propagate with different velocities. A dual receiver detection scheme in which the received waveforms at two different locations are cross-correlated offers a number of distinct advantages over ordinary auto-correlation in detection of signals having traversed a multi-velocity medium. This Note describes some of these advantages, particularly in terms of a duo-velocity medium in which noise and signal propagate at two different velocities, and where the problem is one of passive detection, communications, or measurement of signal or channel characteristics. Previous examination[1-8] of dual receivers has not focussed on a velocity-differential environment, and phased-array studies have used diversity combining drawing upon addition of many receiver outputs rather than correlation of two.

As is well known the seismic channel supports multi-velocity propagation modes[*,9-11], and this could be of special interest in detection of underground nuclear explosions or earthquakes or in differentiating the two (even for simultaneous occurrence)[12-15], in underground communications, and in signal probing to study the Earth structure. Interest may center on differentiating between two aperiodic or two stochastic waveforms, or on detecting a signal imbedded in the natural microseismic noise of the Earth.

Perhaps underwater sound propagation and electromagnetic waves in the ionospheric channel exhibit similar anomalies in velocity of propagation. This paper treats the properties of the dual-receiver detection scheme, without further reference to the specific propagation media to which it can be applied.

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[*] As described in some of the references, the principal types of propagation modes (having different velocities) in the seismic channel are the <u>compressional</u> (P) waves and the <u>shear</u> (S) waves in the body, and the Love and Rayleigh surface waves, the Rayleigh accounting for much of the microseismic noise. Some of the discussion of this paper, e.g., that dealing with direction finding, assumes a somewhat idealized channel model in which the propagation characteristics for each mode are assumed essentially invariant throughout the propagation path. However, the main discussion concerning detection assumes only that the propagation characteristics remain invariant in the region between the two component receivers of the dual receiver, and the velocity components colinear with the axis of the dual-receiver dipole are the ones of interest. Actually, even if this latter invariance did not exist, the major benefits of the relative correlation shift would still arise.

The problem of interest is one in which we wish to detect, at the receiver, the presence of signal (and noise), as compared with just noise, where the noise is microseismic or any undesired signal. The resulting overall cross-correlation function at the output of the dual receiver consists of the signal auto-correlation function, the noise auto-correlation function displaced from the former, and finite-time correlation noise. The advantages derived with the dual-receiver all arise from the relative shifting of the signal and noise correlation functions. The discussion will treat examination of the received waveform at a specified time, at which time the overall correlation function is sampled at the peak of the signal auto-correlation function. In contrast to ordinary auto-correlation, the noise can be separated from the signal term when the noise statistics are not adequately known. Even when the noise statistics are known, there can be a 4 db improvement in effective signal-to-noise ratio over that obtained from auto-correlation. In a passive system where it is not possible to simply increase the transmitter power, this saving may be the final factor which influences whether or not the required detection reliability is achieved.

II. SHIFTED CORRELATION FUNCTIONS

We treat an idealized model to emphasize the method. The dual receiver has the configuration shown in Figure 1. The waveform is received at two points, and the channel between the two receivers is treated as a bi-modal delay line*, having a delay of T_s for the signal and T_n for the noise.

*This paper has been concerned with general conclusions, rather then with fine detail. If there is channel distortion between the two receivers, then of course the expressions for u_z and σ_z^2 must be modified. The intervening channel could then be represented by two delay lines, one each for signal and noise, and each followed by filters having impulse responses respectively of $h_s(t)$ and $h_n(t)$. (These two impulse responses could be the same.)

We then have

$$u_z = \int R_s(\sigma)h_s(\tau - \sigma)d\sigma + \int R_{ns}(\sigma)h_s(\tau - \sigma)d\sigma$$

$$+ \int R_{sn}(\sigma)h_n(\tau - L - \sigma)d\sigma + \int R_n(\sigma)h_n(\tau - L - \sigma)d\sigma.$$

The expressions derived in the Appendix for variance and hence $(S/N)_o$ are similarly altered.

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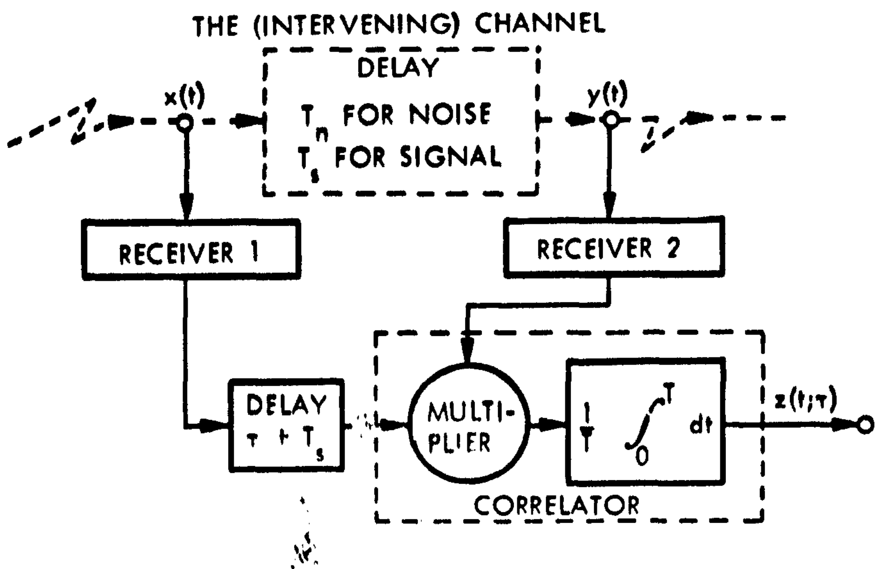

Figure 1. Dual Receiver

(These delays reflect the different velocities for signal and noise). The waveform at receiver 1 is $x(t) = s(t) + n(t)$, where the component terms denote signal and noise respectively. At receiver 2 the waveform is $y(t) = s(t - T_s) + n(t - T_n)$. The detector consists of an artificial delay, and a finite-time correlator. For simplicity in this discussion we examine low-pass correlation (for video waveforms), and the correlator then consists of a multiplier followed by a finite-time integrator. The output of the correlator is sampled for decision at the time at which the signal, if present, has effectively been completely received at the second receiver.

The correlator output can be a random variable[16,17], especially in the presence of noise, and this output consists of the average component (which is the one of interest) and a fluctuation component. (See Appendix). The average output is then $u_z = R_s(\tau) + R_{ns}(\tau) + R_{sn}(\tau + T_s - T_n) + R_n(\tau + T_s - T_n)$, where R_s and R_n are the signal and noise auto-correlation functions, respectively, and the other two terms are the signal and noise cross-correlation functions. When $s(t)$ and $n(t)$ are uncorrelated* we obtain $u_z = R_s(\tau) + R_n(\tau - L)$, where $L = T_n - T_s$, and in particular at $\tau = 0$ this becomes $u_z = R_s(0) + R_n(L)$. We examine the function for $\tau = 0$, and when L is large enough, and hence $R_n(L)$ is small enough, we effectively sample only the signal auto-correlation function. In Figure 2 we show u_z consisting of the signal and the displaced noise auto-correlation functions, and in Figure 3 we show the same for ordinary auto-correlation of the received waveform, corresponding to a zero displacement, $L = 0$.

Normally when autocorrelation is used the noise statistics are also at least partially known, and in particular $R_n(0)$ is known and can be subtracted out from the u_z leaving just $R_s(0)$. Thus examination of $u_z - R_n(0)$ indicates presence or absence of signal, depending upon whether or not this is non-zero (considering, for the moment, the average value, and ignoring the fluctuation component). Thus, if we know $R_n(0)$ then it can be subtracted out anyway, and there is then no direct advantage in obtaining a displacement between the two

*Even when signal and noise are correlated we get proper indication of whether signal is present or not. The function u_z then consists of all four terms indicated above, but as also shown above only the first two terms are significant at $\tau = 0$ when L is large enough. Thus at $\tau = 0$ the significant terms are $R_s(0) + R_{ns}(0)$, and both are present when signal is present and both are absent when signal is absent.

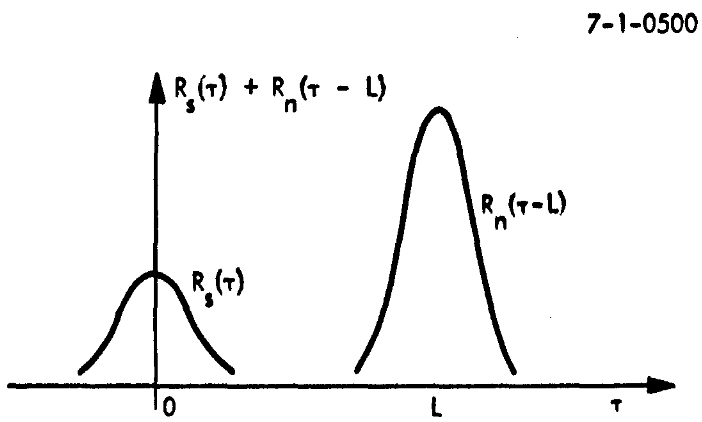

Figure 2. u_z for Displaced Correlation Functions

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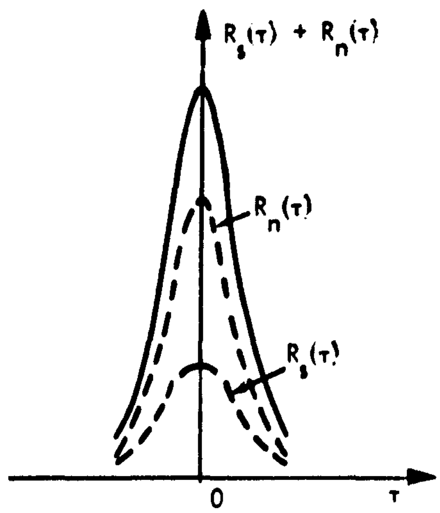

Figure 3. υ_z for $L = 0$, (Ordinary Auto-Correlation)

correlation functions. (There still is however, an indirect advantage resulting in improved signal-to-noise ratio, and this is cited in the next subsection.) However, when the noise statistics are not known and when $R_n(0)$ cannot be subtracted out, there <u>is</u> great merit in being able to achieve a relative displacement in the correlation functions.

The correlation shift is effective with either stochastic or deterministic waveforms. It is of particular value for non-stationary processes, for which the noise term could not ordinarily be subtracted out, the statistics themselves fluctuating. Although we have described the problem for known arrival times, the correlation shift is valuable also for continuous scanning of the waveform, since a new correlation function appears when desired signal is present.

III. SIGNAL-TO-NOISE IMPROVEMENT

The probability of correct decision at the output of the correlator depends, of course, upon the effective signal-to-noise ratio at that point. If we know the noise statistics and can therefore subtract out $R_n(L)$, the signal-to-noise ratio at the correlator output is

$$(S/N)_o = \frac{\left[u_z - R_n(L)\right]^2}{\sigma_z^2} \quad ,$$

where σ_z^2 is the variance of the correlator output. The $(S/N)_o$ is developed in the Appendix for increasing amounts of assumptions and specializations. For stochastic signal and noise which are Gaussian and uncorrelated, and where the noise is white and band-limited at W, and $(S/N)_i$ is small, we obtain

$$(S/N)_o \approx 2\ TW\ (S/N)_i^2 \left[1 + \frac{\sin(4\pi WL)}{(4\pi WL)}\right]^{-1} \quad ,$$

where the subscripts i and o refer respectively to the correlator input and output. Maximum $(S/N)_o$ occurs for $L \approx (3/8W)(257.5/270)$, and it is 4 db better than the value for $L = 0$, the latter corresponding to ordinary autocorrelation. The maximum is essentially satisfied at $L \approx (3/8W)$, and for

known velocity differential this can be achieved simply by setting the separation distance of the receivers as

$$D = L\,|v_s - v_n| \quad ,$$

where v_s and v_n are velocities of propagation.

We note that for L equal to an integer multiple of (1/4W) or for L >> (1/W), the improvement in signal-to-noise ratio over the value for L = 0 is 3 db. We might just remark that for noise band-limited at W, $R_n(L)$ has zeros for L an integer multiple of (1/2W). Then $R_n(L)$ disappears of itself (at $\tau = 0$), even if we don't know the actual values for the noise correlation function. However, we then achieve only the 3 db improvement in signal-to-noise ratio.

IV. CORRELATION, VELOCITY, AND DIRECTIONAL MEASUREMENTS

Although our major interest has focussed on signal detection, various related measurements can be made with the receiver. For example, even if noise and signal are correlated, then knowledge of one correlation function can permit determination of the others. If $R_n(\tau)$ is known, then from samples of the function $u_z = R_s(\tau) + R_{ns}(\tau) + R_{sn}(\tau - L) + R_n(\tau - L)$ around $\tau = L$, we can determine $R_{sn}(\tau)$, since the first two terms are negligible at $\tau = L$ when L is large enough. From this, $R_{ns}(\tau)$ is known, and then from examination of u_z in the vicinity of $\tau = 0$, we can determine $R_s(\tau)$. For stationary stochastic noise, $R_n(\tau)$ could readily be determined from measurements prior to signal reception.

Signal probing can be used for determination of the velocity-differential characteristics of the medium. For example, an active system can generate a strong signal which may propagate over different modes, and the velocity differential determined. We might note that two sources generating signals which propagate at the same speed but which impinge upon the receiver dipole from different directions give rise to an apparent velocity differential.

For the moment we digress and comment on an application where a dual receiver has of course been used for direction finding and source location, wherein a single source is located as being on a particular hyperbola. In our cases of interest the distance between receivers is small, and a signal

source at a much greater distance is located precisely in direction, since it will be lying essentially on the asymptote of one of the aforementioned hyperbolas, all asymptotes bisecting the axis of the dual receiver. (An assumption here is straight-line propagation and a broad wave front). The direction of the signal source is described by θ, the angle between the perpendicular to the dual receiver axis and the line from its center to the source; by v, the speed of propagation; D, the distance between receivers; and Δ, the delay introduced in the correlation. Then $D/\Delta = v \sin \theta$, and knowing either v or θ, the other can then be determined. With a triple-receiver, both v and θ can be determined.

V. REMARKS

Under certain circumstances the different velocity components could be filtered out with differently oriented or with different types of sensors. However, when the filtering must in essence be done with the data processing rather than with the sensors, the detection procedure described has the values discussed and which are summarized below:

1) There is the relative shift in signal and noise correlation functions, arising for deterministic and for stochastic signals, and particularly effective for signal detection when the noise statistics are unknown. Of special interest is the case where noise (and signal) statistics are non-stationary, and $R_r(0)$ could not be subtracted out. Even aperiodic noise and signal occuring at the same time (e.g., simultaneously occuring earthquake and nuclear explosion) can be separated.

2) There is the improvement in signal-to-noise ratio.

3) Auto-correlation and cross-correlations functions can be determined just knowing one.

4) There is application to velocity determination, structural study, and direction finding.

The duo-velocity discussion extends easily to the multi-velocity case.

APPENDIX

We derive the signal-to-noise relation at the output of the cross-correlator for the dual-receiver detection scheme. The following discussion is developed with increasing specialization, and for any actual problem the degree of generality desired will indicate how far one should go in using these relations. Intermediary steps are included to allow one to use the expressions relevant to one's problem. The derivation here given is for signal and noise processes which are random and stationary. An ensemble averaging is indicated by < >. We will obtain the output signal-to-noise for $\tau = 0$, corresponding to the peak of the signal correlation function.

$$\left(\frac{S}{N}\right)_o = \frac{\left[u_z - R_n(L)\right]^2}{\sigma_z^2} .$$

$$z(t;\tau) = \frac{1}{T} \int_{t-T}^{t} x(t-\tau-T_s)\, y(t) dt.$$

$$u_z(\tau) = \langle z(t;\tau)\rangle = \frac{1}{T} \int_{0}^{T} \langle x(t-\tau-T_s)\, y(t)\rangle\, dt$$

$$= R_s(\tau) + R_{ns}(\tau) + R_{sn}(\tau-L) + R_n(\tau-L).$$

When n(t) and s(t) are uncorrelated, $u_z(\tau) = R_s(\tau) + R_n(\tau-L)$; and in particular, at $\tau = 0$, this is $R_s(0) + R_n(L)$, which we denote u_z.

$$\left[\sigma_z(\tau)\right]^2 = \langle z^2(t;\tau)\rangle - \left[u_z(\tau)\right]^2$$

$$= \frac{1}{T^2} \int_0^T \int_0^T \langle x(t-\tau-T_s)\, y(t) x(\rho-\tau-T_s)\, y(\rho)\rangle\, dt d\rho - \left[u_z(\tau)\right]^2.$$

At $\tau = 0$ this becomes:

$$\left[\sigma_z(0)\right]^2 = \sigma_z^2 = \frac{1}{T^2}\int_0^T\int_0^T < [s(t)+n(t)]\,[s(t)+n(t-L)]\,[s(\rho)+n(\rho)]\,[s(\rho)+n(\rho-L)] > dt\,d\rho - u_z^2$$

Now assuming s(t) and n(t) to be Gaussian with zero mean, thereby permitting use of the relation $<x_1x_2x_3x_4> = <x_1x_2><x_3x_4> + <x_1x_3><x_2x_4> + <x_1x_4><x_2x_3>$, and further assuming s(t) and n(t) are independent (uncorrelated), thereby allowing us to discard a greater number of terms, and making use of the relation

$$\frac{1}{T^2}\int_0^T\int_0^T R(\gamma-\rho)\,d\gamma\,d\rho = \frac{2}{T}\int_0^T \left(1 - \frac{\alpha}{T}\right) R(\alpha)\,d\alpha,$$ we obtain*

$$\sigma_z^2 = \frac{2}{T}\int_0^T \left(1 - \frac{\alpha}{T}\right) R_s^2(\alpha)\,d\alpha + \frac{2}{T}\int_0^T \left(1 - \frac{\alpha}{T}\right)\left[R_s(\alpha) + R_n(\alpha)\right]^2 d\alpha$$

$$+ \frac{2}{T}\int_0^T \left(1 - \frac{\alpha}{T}\right) R_n(\alpha+L)R_n(\alpha-L)\,d\alpha + \frac{2}{T}\int_0^T \left(1 - \frac{\alpha}{T}\right) R_s(\alpha)\left[R_n(\alpha-L) + R_n(\alpha+L)\right] d\alpha.$$

Let us consider the case in which $R_s(\alpha)$ and $R_n(\alpha) \to 0$ for $\alpha > A << T$. This permits dropping the $(1 - \alpha/T)$ for the first two integrals. If $L > A$, then the third integral vanishes. We are interested in retaining the last two integrals; therefore, let us treat the case where L is not too large, say $L < A$. Then $(1 - \alpha/T)$ may be dropped completely. Keeping these modifications in mind, we can let the upper limit be infinity with negligible change in the result. Further, since the integrands are even, we can express the integrals over the entire line. With these changes the noise is

*When $L = 0$, corresponding to auto-correlation, σ_z^2 becomes

$$\frac{4}{T}\int_0^T \left(1 - \frac{\alpha}{T}\right)\left[R_s(\alpha) + R_n(\alpha)\right]^2 d\alpha.$$

$$\sigma_z^2 = \frac{1}{T}\int_{-\infty}^{\infty} R_s^2(\alpha)d\alpha + \frac{1}{T}\int_{-\infty}^{\infty}\left[R_s(\alpha) + R_n(\alpha)\right]^2 d\alpha$$

$$+ \frac{1}{T}\int_{-\infty}^{\infty} R_n(\alpha+L)\, R_n(\alpha-L)d\alpha + \frac{1}{T}\int_{-\infty}^{\infty} R_s(\alpha)\left[R_n(\alpha-L) + R_n(\alpha+L)\right] d\alpha.$$

The Fourier Transforms of $R_s(\rho)$ and $R_n(\rho)$ are, respectively, $G(f)$, the signal power spectral density, and $N(f)$, the noise power spectral density.

$$\sigma_z^2 = \frac{2}{T}\int_{-\infty}^{\infty} G^2(f)df + \frac{1}{T}\int_{-\infty}^{\infty} N^2(f)\,[1 + \cos\,(4\pi Lf)]\,df$$

$$+ \frac{2}{T}\int_{-\infty}^{\infty} G(f)\,N(f)\,[1 + \cos\,(2\pi Lf)]\,df.$$

For signal and for white noise (of density N_o) both band-limited at W;

$$\sigma_z^2 = \frac{2}{T}\int_{-W}^{+W} G^2(f)df + \frac{2WN_o^2}{T}\left[1 + \frac{\sin\,(4\pi WL)}{4\pi WL}\right]$$

$$+ \frac{2N_o}{T}\left[R_s(0) + R_s(L)\right].$$

When $G(f) << N_o$, the first term is negligible.*

In terms of the input signal-to-noise ratio, $(S/N)_i = R_s(0)/2WN_o$, the output signal-to-noise ratio is

*For regular cross-correlation detection, the second term is not there and the $R_s(L)$ is not there, leaving $2N_oR_s(0)/T$. For regular auto-correlation, corresponding to L = 0, we have

$$4WN_o^2/T + 4N_oR_s(0)/T \text{ as the value for } \sigma_z^2.$$

$$\left(\frac{S}{N}\right)_o = \frac{2TW\left(\frac{S}{N}\right)_i^2}{1 + \frac{\sin(4\pi WL)}{4\pi WL} + 2\left(\frac{S}{N}\right)_i\left[1 + R_s(L)/R_s(0)\right]} .$$

Actually when $G(f) << N_o$ ----- say by at least several db; e.g., when $(S/N)_i$ is perhaps -10db ----- the last noise term isn't important either. Then

$$\sigma_z^2 = \frac{2WN_o^2}{T}\left[1 + \frac{\sin(4\pi WL)}{4\pi WL}\right]$$

and

$$\left(\frac{S}{N}\right)_o = 2TW\left(\frac{S}{N}\right)_i^2\left[1 + \frac{\sin(4\pi WL)}{4\pi WL}\right]^{-1}$$

Maximum $(S/N)_o$ occurs for $L = (3/8W)(257.5/270)$. This is 4 db better than the value for $L = 0$. With $L = (3/8W)$, corresponding to an angle of 270 degrees in the sinc function above, there is negligible reduction in $(S/N)_o$ from the maximum. We note that for L equal to an integer multiple of (1/4W) and for $L = \infty$, the improvement in signal-to-noise ratio over the value for $L = 0$ is 3 db.

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